1 Rational Numbers

Key Concepts

1. Rational numbers are numbers in the form of $\frac{a}{b}$, where a and b are integers and $b \neq 0$.
2. Rational numbers are closed under the operations of addition, subtraction and multiplication.
3. If we exclude 0, the collection of all other rational numbers is closed under division.
4. The operations of addition and multiplication for rational numbers are

   (a) commutative: i.e., $\frac{-5}{7} + \frac{2}{9} = \frac{-45 + 14}{63} = \frac{-31}{63}$; $\frac{2}{9} + \left(\frac{-5}{7}\right) = \frac{14 + (-45)}{63} = \frac{-31}{63}$

   $\frac{-5}{7} \times \frac{2}{9} = \frac{-10}{63}$; $\frac{2}{9} \times \left(\frac{-5}{7}\right) = \frac{-10}{63}$

   (b) associative: i.e., $\left(\frac{-2}{5} + \frac{1}{5}\right) + \frac{2}{5} = \frac{-2+1}{5} + \frac{2}{5} = \frac{-1}{5} + \frac{2}{5} = \frac{1}{5}$; $\frac{-2}{5} + \left(\frac{2}{5} + \frac{1}{5}\right) = \frac{-2}{5} + \frac{3}{5} = \frac{1}{5}$

   $\left(\frac{-2}{5} \times \frac{1}{5}\right) \times \frac{2}{5} = \left(\frac{-2}{25}\right) \times \frac{2}{5} = \frac{-4}{125}$; $\frac{-2}{5} \times \left(\frac{1}{5} \times \frac{2}{5}\right) = \left(\frac{-2}{5}\right) \times \frac{2}{25} = \frac{-4}{125}$
5. The operations of subtraction and division for rational numbers are

   (a) not commutative: i.e., $\frac{2}{5} - \frac{3}{4} = \frac{8-15}{20} = \frac{-7}{20}$; $\frac{3}{4} - \frac{2}{5} = \frac{15-8}{20} = \frac{7}{20}$

   $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{8}{15}$; $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$

   (b) not associative: i.e., $\frac{1}{5} - \left(\frac{4}{5} - \frac{2}{5}\right) = \frac{1}{5} - \frac{2}{5} = \frac{-1}{5}$; $\left(\frac{1}{5} - \frac{4}{5}\right) - \frac{2}{5} = \frac{-3}{5} - \frac{2}{5} = \frac{-5}{5} = -1$

   $\frac{1}{5} \div \left(\frac{4}{5} \div \frac{2}{5}\right) = \frac{1}{5} \div \left(\frac{\cancel{4}^{2}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}}\right) = \frac{1}{5} \div 2 = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

   $\left(\frac{1}{5} \div \frac{4}{5}\right) \div \frac{2}{5} = \left(\frac{1}{\cancel{5}} \times \frac{\cancel{5}}{4}\right) \times \frac{5}{2} = \frac{1}{4} \times \frac{5}{2} = \frac{5}{8}$
6. Rational number 0 is the additive identity for rational numbers.
7. Rational number 1 is the multiplicative identity for rational numbers.
8. The additive inverse of the rational number $\frac{a}{b}$ is $-\frac{a}{b}$ and vice-versa.
9. The reciprocal or multiplicative inverse of the rational number $\frac{a}{b}$ is $\frac{b}{a}$. So, $\frac{a}{b} \times \frac{b}{a} = 1$.
10. Rational number 0 has no reciprocal or multiplicative inverse.
11. For all rational numbers a, b and c

    $\left.\begin{array}{l} a \times (b + c) = a \times b + a \times c \\ a \times (b - c) = a \times b - a \times c \end{array}\right\}$ Distributive property
12. Rational numbers can be represented on a number line.
13. There are infinite rational numbers between any two given rational numbers.

2 Linear Equations in One Variable

Key Concepts

1. An equation is a statement of equality. An equation must contain equality sign (=) and an expression on either side of it.
   For example, $2y - 4 = 10$ is an equation.

2. In an equation, value of the expression on one side of the equality sign is equal to the value of the expression on the other side.

3. Expressions which form the equation, contain only one variable and the largest exponent of the variable part of any term on either side of the equation is one, such equations are called linear equations.
   For example, $2a - 3 = 7 - 5a$ is linear equation in variable 'a'.

4. The value of the variable in an equation which satisfies the equation is called a solution of the equation.

5. A linear equation may have its solution as rational number.

6. Just as numbers, variables, can also be transposed from one side of the equation to the other.

   For example,
   $$\begin{aligned} 3y - 5 &= 2y + 8 \\ \Rightarrow \quad 3y &= 2y + 8 + 5 \\ \Rightarrow \quad 3y - 2y &= 13 \\ \Rightarrow \quad y &= 13 \end{aligned}$$

7. The utility of linear equations is in their diverse applications: different problems based on numbers, age, perimeter, etc.

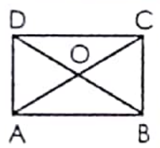
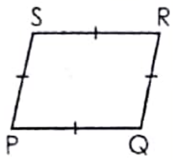
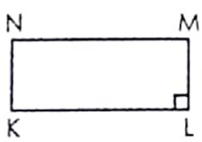
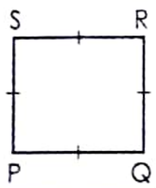
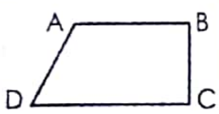
3 Understanding Quadrilateral

Key Concepts

1. A simple closed curve made up of line segments only is called a polygon.
2. Each common end point (intersection) is called a vertex and each segment is called a side of the polygon.
3. A diagonal of a polygon is a line segment whose end points are two non-consecutive vertices.
4. A polygon is convex, if line segment joining any two points in the interior of the polygon lies in the interior of the polygon. A polygon that is not convex is concave.
5. A polygon whose all sides and all angles are equal is called a regular polygon. A regular polygon is both equiangular and equilateral.
6. Polygons are named according to their number of sides such as.

Number of sides	4	5	6	7	8	9	10
Name of the polygon	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	Nonagon	Decagon

7. Any polygon having four sides is called a quadrilateral.
8. Quadrilaterals are classified according to their properties.

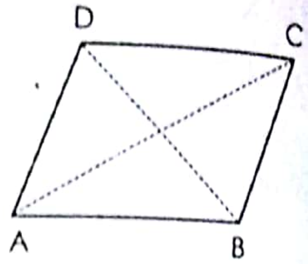
Quadrilaterals	Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel. 	(i) Opposite sides are equal. (ii) Opposite angles are equal. (iii) Diagonals bisect each other.
Rhombus: A parallelogram with all the sides of equal length. 	(i) All the properties of a parallelogram. (ii) Diagonals are perpendicular to each other.
Rectangle: A parallelogram with a right angle. 	(i) All the properties of a parallelogram. (ii) Each angle is a right angle. (iii) Diagonals are equal.
Square: A rectangle with sides of equal length. 	All the properties of a parallelogram, rhombus and a rectangle.
Trapezium: A quadrilateral with one pair of opposite sides parallel. 	(i) Bases of the trapezium are parallel. (ii) No sides, angles and diagonals are congruent.

4 Practical Geometry

Key Concepts

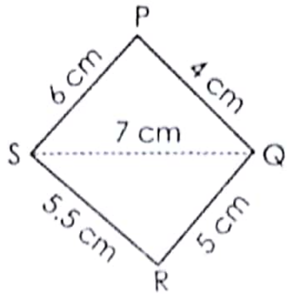
1. A quadrilateral has four sides, four angles and two diagonals.

2. ABCD is a quadrilateral.
   AB, BC, CD and DA are its four sides.
   AC and BD are its diagonals.
   ∠DAB, ∠ABC, ∠BCD and ∠CDA are its four angles.

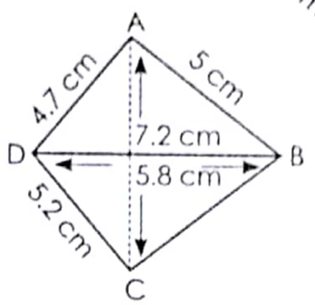


3. A quadrilateral can be constructed uniquely, if any of five of its components are given, such as:

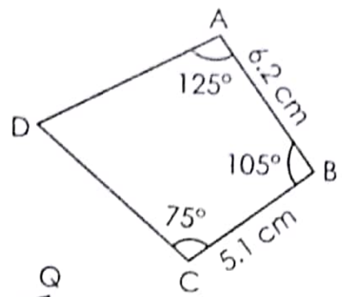
   (i) four sides and a diagonal.
   Given; PQ, QR, RS and PS – sides
   SQ – diagonal



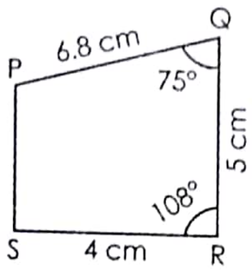
   (ii) three sides and two diagonals.
   Given; AB, AD and CD – sides
   AC and BD – diagonals



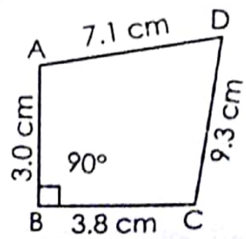
   (iii) three angles and two included sides.
   Given; ∠ABC, ∠BCD and ∠DAB – angles
   AB and CB – sides



   (iv) three sides and two angles.
   Given; PQ, QR and RS – sides
   ∠PQR and ∠QRS – angles



   (v) four sides and one angle.
   Given; AB, BC, CD and DA – sides
   ∠ABC – angle



6 Squares and Square Roots

Key Concepts

- A number 'n' is a perfect square, if $n = m^2$ for some integer m.
- All squared numbers end with 0, 1, 4, 5, 6 or 9 at units place.
- A number having 2, 3, 7 or 8 at units place is never a perfect square.
- The number of zeroes at the end of a perfect square is always even.
- A number ending in an even number of zeroes may or may not be a perfect square. For example, 400 is a perfect square whereas 2900 is not a perfect square.
- Squares of even numbers are always even numbers and squares of odd numbers are always odd numbers.

  For example, $2^2 = 4$, $4^2 = 16$, etc.

  and $1^2 = 1$, $3^2 = 9$, etc.
- The difference of squares of two consecutive natural numbers is equal to their sum.

  For example, $2^2 - 1^2 = 4 - 1 = 3 = 2 + 1 = 3$

  $\Rightarrow$ $2^2 - 1^2 = 2 + 1$

  Also, $3^2 - 2^2 = 9 - 4 = 5 = 3 + 2$

  $\Rightarrow$ $3^2 - 2^2 = 3 + 2$
- The square of a natural number 'n' is equal to the sum of first 'n' odd natural numbers.

  For example, $4^2 = 1 + 3 + 5 + 7 = 16$
- A triplet (m, n, p) of three numbers m, n and p is called a Pythagorean triplet, if $m^2 + n^2 = p^2$.
- A number 'a' is a square root of a number 'b', if $a^2 = b$. So, square root is the inverse operation of the operation of finding square.
- The symbol $\sqrt{\ }$ is used for the positive square root of a number.

  For example, $4 \times 4 = 16$

  So, $\sqrt{16} = 4$.
- There are two integral square roots of a perfect square number.
- Square root of a square number can be found by using
  - (i) method of repeated subtraction of odd numbers 1, 3, 5, ..., .
  - (ii) by prime factorisation method.
  - (iii) long division method.
- If a perfect square is of n digits, then its square root will have $\frac{n}{2}$ digits (if n is even) and $\frac{n+1}{2}$ digits (if n is odd).

7 Cubes and Cube Roots

Key Concepts

1. The number obtained when a number is multiplied by itself three times is known as cube of the number.
   For example, cube of $4 = 4^3 = 4 \times 4 \times 4 = 64$
2. A number is said to be a perfect cube, if it is the cube of some integer.
   For example, $1^3 = 1$, $2^3 = 8$, $3^3 = 27$ and so on. Each of 1, 8 and 27 are perfect cubes.
3. A given natural number is a perfect cube, if it is expressible as the product of triplets of equal factors.
   For example, 729 can be resolved as prime factors

9	729
9	81
9	9
	1

   Now, $729 = 9 \times 9 \times 9 = 9^3$
   Thus, 729 is a perfect cube and it is cube of 9.
4. Cubes of all even numbers are even and cubes of all odd numbers are odd.
5. Cubes of the numbers ending in digits 1, 4, 5, 6 and 9 are the numbers ending in the same digit.
6. Cubes of the numbers ending with digits 2 and 8 end with digits 8 and 2 respectively.
7. Cubes of the numbers ending in digits 3 and 7 end in digits 7 and 3 respectively.
8. Cubes of negative integers are negative and cubes of positive integers are positive.
9. If a number ends in a zero, its cube will end in three zeros.
10. The symbol '$\sqrt[3]{}$' denotes cube root.
11. A number 'm' is a cube root of a number n, if $n = m^3$. If m is a cube root of n, we write $m = \sqrt[3]{n}$.
12. For a positive integer 'x', $(-x)^3 = -x^3$, i.e., $\sqrt[3]{-x^3} = -x$.
13. If 'x' and 'y' ($\neq 0$) are perfect cubes, then $\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$.

8 Comparing Quantities

Key Concepts

1. Per cent means 'per hundred'. Per cent is the ratio of a number to 100.

2. Per cent increase/decrease = $\frac{\text{Change in amount}}{\text{Original amount}} \times 100\%$.

3. Profit% = $\frac{\text{Profit}}{\text{Cost price}} \times 100\%$.

4. Loss% = $\frac{\text{Loss}}{\text{Cost price}} \times 100\%$.

5. Discount is a reduction given in marked price. i.e.,

   Discount = Marked price (MP) – Selling price (SP).

6. Discount% = $\frac{\text{Discount}}{\text{Marked price}} \times 100\%$.

7. Additional expenses made after buying an article are included in the cost price and are known as overhead expenses.

   Cost price (CP) = Buying price + Overhead expenses

8. Goods and service tax (GST) is a tax levied upon selling prices of the articles.

9. GST = tax% of bill amount.

10. Compound interest is the interest calculated on the previous year's amount.

11. Compound Interest = Amount – Principal.

12. When the interest is compounded annually, then

    $$A = P\left(1 + \frac{R}{100}\right)^n$$

    Here; A = amount, P = principal, R = rate of interest and n = time period

13. When the interest is compounded half yearly, then

    $$A = P\left(1 + \frac{R}{200}\right)^{2n}$$

    Here; $\frac{R}{2}$ = half yearly rate and 2n = number of half years